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# The Crystal Structures of $\mathbf{P}_{4} \mathbf{S}_{10}$ and $\mathbf{P}_{4} \mathbf{S}_{7}$ 

By Aafje Vos and E. H. Wiebenga<br>Laboratorium voor Anorganische en Physische Chemie der Rijksuniversiteit Groningen, The Netherlands

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#### Abstract

$\mathrm{P}_{4} \mathrm{~S}_{10}$ and $\mathrm{P}_{4} \mathrm{~S}_{7}$ belong to the space groups $P \overline{1}$ and $P 2_{1} / n$ with 2 and 4 molecules per unit cell respectively. Approximate coordinates were found from three-dimensional Patterson syntheses; the coordinates were refined by computing Fourier syntheses of projections. The structures consist of molecules of $\mathrm{P}_{4} \mathrm{~S}_{10}$ and $\mathrm{P}_{4} \mathrm{~S}_{7}$. The intermolecular distances are approximately $3 \cdot 4 \AA$. In both molecules two types of $\mathrm{P}-\mathrm{S}$ bonds may be distinguished with bond lengths of 2.08 and $1.95 \AA$ respectively. The e.s.d. is approximately $0.01 \AA$ for the former and $0.01_{5} \AA$ for the latter. The symmetry of the $P_{4} S_{10}$ molecule is $\overline{4} 3 \mathrm{~m}$; the $\mathrm{S}-\mathrm{P}-\mathrm{S}$ and $\mathrm{P}-\mathrm{S}-\mathrm{P}$ angles show no significant deviation from $109 \cdot 5^{\circ}$. In $\mathrm{P}_{4} \mathrm{~S}_{7}$ some $\mathrm{S}-\mathrm{P}-\mathrm{S}$ bond angles are significantly different from $109 \cdot 5^{\circ}$; the molecule has the symmetry mm 2 and contains a remarkably long $\mathrm{P}-\mathrm{P}$ bond ( $2.37 \AA$ with an e.s.d. of $0.04 \AA$ ).


## Introduction

Phosphorus and sulfur form the compounds $\mathrm{P}_{4} \mathrm{~S}_{3}$, $\mathrm{P}_{4} \mathrm{~S}_{5}, \mathrm{P}_{4} \mathrm{~S}_{7}$ and $\mathrm{P}_{4} \mathrm{~S}_{10}$ (Stock, $1910 a, b, c$; Treadwell \& Beeli, 1935; Pernert \& Brown, 1949). The only direct information available on the structures of these compounds was an electron-diffraction study of $\mathrm{P}_{4} \mathrm{~S}_{3}$ (Hassel \& Pettersen, 1941). For $\mathrm{P}_{4} \mathrm{~S}_{10}$ molecules in solution a structure was postulated (Pernert \& Brown, 1949) analogous to that of $\mathrm{P}_{4} \mathrm{O}_{10}$ molecules, as determined by Hampson \& Stosick (1938) by electron diffraction. Though this structure for the molecule in solution is very probable, this does not imply that solid $\mathrm{P}_{4} \mathrm{~S}_{10}$ should also consist of these molecules. For $\mathrm{P}_{4} \mathrm{O}_{10}$, for example, three solid modifications were reported, only one form containing molecules of $\mathrm{P}_{4} \mathrm{O}_{10}$ (de Decker \& MacGillavry, 1941; de Decker, 1941; MacGillavry, de Decker \& Nijland, 1949). The structures proposed for $\mathrm{P}_{4} \mathrm{~S}_{7}$ and $\mathrm{P}_{4} \mathrm{~S}_{5}$ (Pernert \& Brown, 1949) are far less certain than that suggested for $\mathrm{P}_{4} \mathrm{~S}_{10}$ and did not find general acceptance.

Our X-ray examination of crystalline $P_{4} S_{10}$ and $\mathrm{P}_{4} \mathrm{~S}_{7}$ shows these compounds to consist of molecules of $P_{4} S_{10}$ and $P_{4} S_{7}$. The structure reported for the $\mathrm{P}_{4} \mathrm{~S}_{10}$ molecule confirms that proposed by Pernert \& Brown; that observed for $\mathrm{P}_{4} \mathrm{~S}_{7}$ is different from all structures postulated previously for this compound.

The crystal structures of $P_{4} \mathrm{~S}_{5}$ and $\mathrm{P}_{4} \mathrm{~S}_{3}$ are now being investigated in our laboratory.

## Materials

$\mathrm{P}_{4} \mathrm{~S}_{10}$ and $\mathrm{P}_{4} \mathrm{~S}_{7}$ were prepared by melting a mixture of phosphorus and sulfur (Organic Synth., 1932). After solidification of the melt, crystals could be obtained by a slow continuous extraction with $\mathrm{CS}_{2}$. As the crystals are not stable in moist air, they were transferred into a boron silicate glass capillary in which they could be kept for a long time. Perpendicular to the crystal axis, about which the X-ray photographs were made, their diameter was approximately 0.1 mm .

## $\mathrm{P}_{4} \mathrm{~S}_{10}$

Unit cell and space group
Optical goniometric measurements and a construction of the reciprocal lattice from oscillation, rotation and Weissenberg photographs about the $a$ axis showed the crystals to be triclinic. With $\lambda(\mathrm{Cu} K \alpha)=1.5418 \AA$ the following lattice constants were obtained:

$$
\begin{aligned}
a=9 \cdot 07, b & =9 \cdot 18, c=9 \cdot 19 \AA ; \\
\alpha & =92 \cdot 4, \beta=101 \cdot 2, \gamma=110 \cdot 5^{\circ} .
\end{aligned}
$$

With a density of $2.09 \mathrm{g.cm} .^{-3}$ (Stock, $1910 c$ ) the number of molecules per cell was calculated to be 1.98 . Wilson's statistical method (Wilson, 1949) and the absence of piezoelectricity indicated the space group $P \overline{1}$, which was confirmed during the structure analysis.

## Structure factors

The intensities of most reflexions $h k l$ were measured on integrated oscillation photographs (Wiebenga \& Smits, 1950; Boswijk, 1954, p. 35) about the $a$ axis, taking an inclination angle of $20^{\circ}$. The photographs were taken with Zr-filtered Mo radiation, using three films of different speed during each exposure. The oscillation range was $20^{\circ}$ with an overlap of $5^{\circ}$ for two successive photographs. The indices of the reflexions $h k l$ were determined graphically from the reciprocal lattice. The intensities of the reflexions on different photographs were put on the same scale by comparing the intensities of the reflexions which successive photographs had in common. The maximum deviation of these scaling factors from those derived from the exposure times of the different photographs was $14 \%$.
No absorption correction had to be applied, because of the small crystals used. A correction for oblique incidence was made (Cox \& Shaw, 1930) as well as a correction for the Lorentz and polarization factors. Use was made of a general expression for the Lorentz factor (Boswijk, 1954, p. 36). The relative intensities of approximately 900 reflexions were measured. They were put on an approximate absolute scale using Wilson's statistical method (Wilson, 1949). The scaling factor thus obtained later appeared to show a difference of approximately $9 \%$ from the final one, derived from a comparison of observed and calculated structure factors.

## Determination of the approximate structure

With the $F^{2}$ values obtained, a three-dimensional Patterson synthesis was computed. This vector map shows an interesting regular arrangement of high peaks. At a distance of about $2 \AA$ the origin is tetrahedrically surrounded by only four peaks, $A-D$ (Fig. 1), of a weight corresponding to eight coincident atomic distances. This suggests that the eight phosphorus atoms in the unit cell are tetrahedrically sur-


Fig. 1. Some high peaks from the three-dimensional Patterson synthesis of $P_{4} S_{10}$, shown schematically in [010] projection.
rounded by sulfur atoms and that these $\mathrm{PS}_{4}$ tetrahedra are either parallel or centrosymmetrically oriented. A centrosymmetrical arrangement of two $\mathrm{P}_{4} \mathrm{~S}_{10}$ molecules of a general shape shown in Fig. 2


Fig. 2. Molecule of $\mathrm{P}_{4} \mathrm{~S}_{10}$.
satisfies this condition. Such a structure would lead to additional peaks in the Patterson map of weights corresponding to ten coincident atomic distances and at positions corresponding to the edges of the $\mathrm{PS}_{4}$ tetrahedra. Since these peaks were actually observed in the Patterson synthesis (peaks $E-J$ in Fig. 1) a structure of $\mathrm{P}_{4} \mathrm{~S}_{10}$ molecules was assumed. The orientation of these molecules could be taken from the vector map; the approximate location relative to the centre of symmetry was determined by trial and error making use of the reflexions $h 00,0 k 0$ and $00 l$.

## Refinement of the structure

[010] projection.-With the approximate coordinates, the structure factors for the reflexions $h 0 l$ were calculated. The agreement between $F_{c}$ and $F_{o}$ was sufficient to determine the signs of 72 out of the 74 observed structure factors. With these structure factors a Fourier synthesis of the [010] projection was computed. The fourteen (independent) atoms appeared to be fairly well resolved in this projection and it was possible to determine their $x$ and $z$ coordinates from the contour map. These coordinates were refined by consecutive Fourier syntheses and corrected for series termination by Booth's backshift method (Booth, 1946). The final electron-density map is shown in Fig. 3(a). The final value of the disagreement index $R \equiv \Sigma_{b l}| | F_{o}\left|-\left|F_{c}\right| \div \Sigma_{h l}\right| F_{o} \mid$ is $0 \cdot 11$, including the observed reflexions only.* Temperature factor:

$$
\exp \left(-2 \cdot 36 \sin ^{2} \theta / \lambda^{2}\right) .
$$

[^0]

Fig. 3. (a) $\mathbf{P}_{4} \mathrm{~S}_{10}$ : final electron-density projection along [010] on the (010) plane. Contours at intervals of 6 e./1.08 $\AA^{2}$, the 3 e./ $1.08 \AA^{2}$ line being broken.
(b) $\mathrm{P}_{4} \mathrm{~S}_{10}$ : final electron-density projection along [100] on the (100) plane. Contours at intervals of $6 e . / 1 \cdot 10 \AA^{2}$, the 3 e. $/ 1 \cdot 10 \AA^{2}$ line being broken.
[100] projection.-Starting from the approximate structure, the signs of 64 out of the 76 observed reflexions could be determined. The coordinates were refined in the same way as reported for the [010] projection. Fig. 3(b) shows the final electron-density map; $R=0 \cdot 11$.* Temperature factor:

$$
\exp \left(-2 \cdot 36 \sin ^{2} \theta / \lambda^{2}\right) .
$$

[001] projection.-Since only three atoms were well resolved in this projection a refinement of the coordinates by the Fourier method was not attempted. Using the coordinates and the temperature factor as determined from the other projections, a disagreement index $R=\cdot 0 \cdot 12$ was calculated for the observed $h k 0$ reflexions.*

The final values for the atomic coordinates are listed in Table 1.

## Accuracy of the atomic coordinates

An estimate of the accuracy of the final coordinates was obtained by comparing the $14 z$ coordinates as determined independently from the [010] and [100] projections respectively. Their r.m.s. deviation is $0.026 \AA$, corresponding to $\sigma(z)=0.018 \AA$. This is approximately twice as large as the r.m.s. average for

[^1]Table 1. $\mathrm{P}_{4} \mathrm{~S}_{10}$ : final atomic coordinates
(a) [100] projection

|  | $y$ | $z$ |
| :--- | ---: | ---: |
|  | $\overbrace{1}$ | 0.087 |
| $\mathrm{P}_{1}$ | -0.190 |  |
| $\mathrm{P}_{2}$ | 0.153 | -0.289 |
| $\mathrm{P}_{3}$ | 0.435 | -0.297 |
| $\mathrm{P}_{4}$ | 0.345 | 0.034 |
| $\mathrm{~S}_{1}$ | -0.074 | -0.195 |
| $\mathrm{~S}_{2}$ | -0.014 | -0.285 |
| $\mathrm{~S}_{3}$ | 0.655 | -0.308 |
| $\mathrm{~S}_{4}$ | 0.176 | 0.033 |
| $\mathrm{~S}_{5}$ | 0.065 | -0.374 |
| $\mathrm{~S}_{6}$ | 0.343 | -0.405 |
| $\mathrm{~S}_{7}$ | 0.245 | -0.066 |
| $\mathrm{~S}_{8}$ | 0.609 | -0.407 |
| $\mathrm{~S}_{9}$ | 0.526 | -0.079 |
| $\mathrm{~S}_{10}$ | 0.430 | 0.237 |

(b) [010] projection

| $x$ | $z$ |
| :---: | :---: |
| $0 \cdot 259$ | -0.187 |
| -0.090 | -0.288 |
| 0.267 | -0.297 |
| 0.178 | 0.037 |
| $0 \cdot 357$ | -0.197 |
| 0.008 | -0.284 |
| $0 \cdot 367$ | -0.308 |
| $0 \cdot 288$ | 0.030 |
| -0.316 | -0.372 |
| 0.024 | -0.403 |
| -0.062 | -0.062 |
| $0 \cdot 375$ | -0.406 |
| $0 \cdot 298$ | $-0.080$ |
| 0.199 | 0.245 |

Final $z$ coordinates are derived from (a) and (b) by taking the average values.
the standard deviations calculated for the $z$ coordinates of the different atoms by applying Cruickshank's formula (Cruickshank, 1949). This discrepancy is probably due to incomplete corrections for series termination. The backshifts were rather large $(0.03 \AA$ on an average) because of the partial overlap of some atoms and the lack of some fairly weak reflexions which could not be measured on the oscillation photographs; moreover, they were, especially for some atoms, very sensitive to small variations of the coordinates on which the $F_{c}$ syntheses were based. The 'experimental' standard deviation $\sigma=0.018 \AA$ was assumed for all coordinates. For the atomic distances this corresponds to a standard deviation $\sigma(d)=V 2 \times 0.018=0.026 \AA$, resulting from inaccuracies in the intensities and imperfections of the backshift corrections. Apart from these errors, the atomic distances are subject to errors due to inaccuracies in the unit-cell dimensions and angles. Their influence was estimated to be represented by a standard deviation of approximately $0.02 \AA$ in the atomic distances. The total standard deviation in the bond lengths thus amounts to approximately $\left(0.026^{2}+0.02^{2}\right)^{\frac{1}{2}}=0.03 \AA$. The corresponding standard deviation in the bond angles (Cruickshank \& Robertson, 1953) is $1 \cdot 2^{\circ}$.

## Discussion

The structure consists of molecules of $\mathrm{P}_{4} \mathrm{~S}_{10}$ separated by shortest intermolecular distances of approximately $3 \cdot 4 \AA$. The [010] and [100] projections are shown in Fig. 4. The molecule is shown in Fig. 2; its bond lengths and angles are listed in Table 2. Since no distinction between phosphorus and sulfur atoms could be made from the Fourier maps, their distinction is entirely based on chemical arguments. From Table 2 and Fig. 2 it is seen that the P-S distances range from 2.02 to $2.13 \AA$ when the sulfur atoms are connected to two phosphorus atoms, and from 1.91 to $1.98 \AA$ when they are connected to one phosphorus atom only.


Fig. 4. (a) $\mathrm{P}_{4} \mathrm{~S}_{10}$ : projection along [010] on the (010) plane. (b) $\mathrm{P}_{4} \mathrm{~S}_{10}$ : projection along [100] on the (100) plane.

Table 2. $\mathrm{P}_{4} \mathrm{~S}_{10}$ : intramolecular bond lengths and angles

| $\mathrm{P}_{1}-\mathrm{S}_{1}$ | $1 \cdot 97 \AA$ | $\mathrm{~S}_{1}-\mathrm{P}_{1}-\mathrm{S}_{2}$ | $110^{\circ}$ | $\mathrm{S}_{4}-\mathrm{P}_{4}-\mathrm{S}_{7}$ | $110^{\circ}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{P}_{1}-\mathrm{S}_{2}$ | $2 \cdot 12$ | $\mathrm{~S}_{1}-\mathrm{P}_{1}-\mathrm{S}_{3}$ | 109 | $\mathrm{~S}_{4}-\mathrm{P}_{4}-\mathrm{S}_{9}$ | 107 |
| $\mathrm{P}_{1}-\mathrm{S}_{3}$ | $2 \cdot 07$ | $\mathrm{~S}_{1}-\mathrm{P}_{1}-\mathrm{S}_{4}$ | 111 | $\mathrm{~S}_{4}-\mathrm{P}_{4}-\mathrm{S}_{10}$ | 111 |
| $\mathrm{P}_{1}-\mathrm{S}_{4}$ | $2 \cdot 08$ | $\mathrm{~S}_{2}-\mathrm{P}_{1}-\mathrm{S}_{3}$ | 109 | $\mathrm{~S}_{7}-\mathrm{P}_{4}-\mathrm{S}_{9}$ | 111 |
| $\mathrm{P}_{2}-\mathrm{S}_{2}$ | $2 \cdot 02$ | $\mathrm{~S}_{2}-\mathrm{P}_{1}-\mathrm{S}_{4}$ | 108 | $\mathrm{~S}_{7}-\mathrm{P}_{4}-\mathrm{S}_{10}$ | 108 |
| $\mathrm{P}_{2}-\mathrm{S}_{5}$ | $1 \cdot 91$ | $\mathrm{~S}_{3}-\mathrm{P}_{1}-\mathrm{S}_{4}$ | 110 | $\mathrm{~S}_{9}-\mathrm{P}_{4}-\mathrm{S}_{10}$ | 109 |
| $\mathrm{P}_{2}-\mathrm{S}_{6}$ | $2 \cdot 13$ | $\mathrm{~S}_{2}-\mathrm{P}_{2}-\mathrm{S}_{5}$ | 111 | $\mathrm{P}_{1}-\mathrm{S}_{2}-\mathrm{P}_{2}$ | 110 |
| $\mathrm{P}_{2}-\mathrm{S}_{7}$ | $2 \cdot 13$ | $\mathrm{~S}_{2}-\mathrm{P}_{2}-\mathrm{S}_{6}$ | 113 | $\mathrm{P}_{1}-\mathrm{S}_{3}-\mathrm{P}_{3}$ | 109 |
| $\mathrm{P}_{3}-\mathrm{S}_{3}$ | $2 \cdot 07$ | $\mathrm{~S}_{2}-\mathrm{P}_{2}-\mathrm{S}_{7}$ | 108 | $\mathrm{P}_{1}-\mathrm{S}_{4}-\mathrm{P}_{4}$ | 110 |
| $\mathrm{P}_{3}-\mathrm{S}_{6}$ | $2 \cdot 08$ | $\mathrm{~S}_{5}-\mathrm{P}_{2}-\mathrm{S}_{6}$ | 110 | $\mathrm{P}_{2}-\mathrm{S}_{6}-\mathrm{P}_{3}$ | 107 |
| $\mathrm{P}_{3}-\mathrm{S}_{8}$ | $1 \cdot 98$ | $\mathrm{~S}_{5}-\mathrm{P}_{2}-\mathrm{S}_{7}$ | 107 | $\mathrm{P}_{2}-\mathrm{S}_{7}-\mathrm{P}_{4}$ | 110 |
| $\mathrm{P}_{3}-\mathrm{S}_{9}$ | $2 \cdot 06$ | $\mathrm{~S}_{6}-\mathrm{P}_{2}-\mathrm{S}_{7}$ | 108 | $\mathrm{P}_{3}-\mathrm{S}_{9}-\mathrm{P}_{4}$ | 108 |
| $\mathrm{P}_{4}-\mathrm{S}_{4}$ | $2 \cdot 12$ | $\mathrm{~S}_{3}-\mathrm{P}_{3}-\mathrm{S}_{6}$ | 110 |  |  |
| $\mathrm{P}_{4}-\mathrm{S}_{7}$ | $2 \cdot 04$ | $\mathrm{~S}_{3}-\mathrm{P}_{3}-\mathrm{S}_{8}$ | 109 |  |  |
| $\mathrm{P}_{4}-\mathrm{S}_{9}$ | $2 \cdot 10$ | $\mathrm{~S}_{3}-\mathrm{P}_{3}-\mathrm{S}_{9}$ | 112 |  |  |
| $\mathrm{P}_{4}-\mathrm{S}_{10}$ | $1 \cdot 96$ | $\mathrm{~S}_{6}-\mathrm{P}_{3}-\mathrm{S}_{8}$ | 107 |  |  |
|  |  | $\mathrm{~S}_{6}-\mathrm{P}_{3}-\mathrm{S}_{9}$ | 111 |  |  |
|  |  | $\mathrm{~S}_{8}-\mathrm{P}_{3}-\mathrm{S}_{9}$ | 108 |  |  |

The conclusion may be drawn that two essentially different bond lengths can be distinguished in the molecule: one, with an average value of $2 \cdot 08_{5} \AA$, corresponding to a 'single' $\mathrm{P}-\mathrm{S}$ bond; and another, with a mean value of $1.95_{5} \AA$, corresponding to a 'double' P-S bond. It was shown by a many-parameter significance test (Cruickshank \& Robertson, 1953) that the
observed deviations from this model are not significant. Assuming for all atomic coordinates a standard deviation of $0 \cdot 03 / / 2 \AA$, a value of $15 \cdot 1$ for $T_{o}^{2}$ was obtained corresponding, for 14 degrees of freedom, to $P \approx 0.4$ (Cruickshank \& Robertson's notation is used). If, on the other hand, all bond lengths were supposed to be essentially equal, with an average value of $2.05 \AA$, the deviations from this model were highly significant ( $P<0.001$ ), showing that the latter interpretation of the experimental results is not correct.

The bond angles range from 107 to $113^{\circ}$. The deviations from $109.5^{\circ}$ are not significant. Hence it may be concluded that each phosphorus atom is tetrahedrically surrounded by four sulfur atoms. The averages of the observed bond lengths, $2 \cdot 08_{5}$ and $1 \cdot 95_{5} \AA$, which are subject to an estimated standard deviation of $0.01 \AA$ and $0.01_{5} \AA$ respectively, may be compared with Pauling's values (Pauling, 1945, p. 164) for the lengths of a single ( $2 \cdot 14 \AA$ ) and double ( $1.94 \AA$ ) bond between phosphorus and sulfur.

## $\mathbf{P}_{4} \mathbf{S}_{\mathbf{7}}$ <br> Unit cell and space group

The cell dimensions were obtained from rotation and Weissenberg photographs about the three crystal axes. With $\lambda(\mathrm{Cu} K \alpha)=1.5418 \AA$ and $\lambda(\mathrm{Mo} K \alpha)=0.7107 \AA$, the following values for the lattice constants of the monoclinic crystals were obtained:

$$
\begin{gathered}
a=8.87 \pm 0.02, b=17.35 \pm 0.03, c=6.83 \pm 0.02 \AA \\
\beta=92.7 \pm 0 \cdot 4^{\circ} .
\end{gathered}
$$

With a density of $2 \cdot 19$ g.cm. ${ }^{-3}$ (Stock, $1910 b$ ), the number of molecules per unit cell was calculated to be $4 \cdot 00$. The systematic absences indicate the space group $P 2_{1} / n$.

## Structure factors

The intensities of most reflexions $h k l$ were measured on integrated equi-inclination Weissenberg photographs about the $c$ axis, taken with Ni-filtered Cu radiation (Wiebenga \& Smits, 1950; Smits \& Wiebenga, 1953). In addition to this, integrated Mo data were available for the $0 k l$ and $h k 0$ reflexions. Weak reflexions $0 k l$ and $h k 0$ were estimated visually on ordinary Weissenberg photographs of long exposure time, using both Ni-filtered Cu and Zr -filtered Mo radiation. Use was made of the multiple-film technique, with three films of different speed for the Mo photographs. No absorption correction had to be applied to the Mo data; the Cu data were roughly corrected for absorption. The intensities measured on $c$-axis Weissenberg photographs of different layer lines were correlated by zero-layer-line Weissenberg photographs about the $a$ and $b$ axes.

## Determination of the approximate structure

After some unsuccessful applications of phase relationships between the structure factors $0 k l$ and $h k 0$


Fig. 5. $\mathrm{P}_{4} \mathrm{~S}_{7}$ : final electron-density projection (a) along [100] on the (100) plane, (b) on (010), (c) along [001] on the (001) plane. Contours at intervals of $6 \mathrm{e} . \AA^{-2}$, the 3 e. $\AA^{-2}$ line being broken.
respectively, a three-dimensional Patterson synthesis was computed. It appeared that, at a distance of approximately $2 \AA$, the origin was tetrahedrically surrounded by four somewhat blurred, independent peaks; this was considered as an indication of the presence of approximately parallel $\mathrm{PS}_{4}$ tetrahedra. An interpretation of the Harker section ( $x, \frac{1}{2}, z$ ) was difficult because of the many 'non-Harker maxima' and the ambiguity connected with the presence of two independent screw axes in the unit cell. The information obtained from the section $\left(x, \frac{1}{2}, z\right)$ can be combined however, with that obtained from the zero section $(x, 0, z)$, the Harker line $\left(\frac{1}{2}, y, \frac{1}{2}\right)$, general peaks in the three-dimensional Patterson synthesis and the relative intensities of reflexions $h 0 l$ with $h$ and $l$ even, as was extensively described by Albrecht \& Corey (1939). In this way, keeping in mind the information obtained from the maxima around the origin, the approximate positions of two phosphorus and seven sulfur atoms could be determined. These phosphorus atoms are tetrahedrically surrounded by sulfur atoms, the two tetrahedra having one sulfur atom in common. The coordinates of the remaining two phosphorus atoms were determined less directly, combining chemical evidence with the positions of some Harker maxima which had not been used yet. The molecule is shown in Fig. 7.

## Refinement of the structure

With the approximate coordinates the structure factors of the reflexions $0 k l, h 0 l$ and $h k 0$ were calculated. The agreement between observed and calculated values was sufficient to determine the signs of 61, 39 and 67 out of the 86,51 and 84 observed, independent, structure factors for the [100], [010] and [001] projections respectively. Some atoms proved to be poorly resolved in the projections, as is shown in

Fig. 5. A Fourier refinement of the coordinates was possible by comparing the 'observed' electron-density maps with 'calculated' ones, combining the information from the three projections. The coordinates were corrected for the finite-series effect by the application of Booth's backshift method (Booth, 1946). The final coordinates are listed in Table 3. The disagreement indices $R$ amount to $0.08,0.13$ and 0.10 for the observed $0 k l, h 0 l$ and $h k 0$ reflexions respectively.* Temperature factor: $\exp \left(-2 \cdot 47 \sin ^{2} \theta / \lambda^{2}\right)$.

## Accuracy of the atomic coordinates

The effect of inaccuracies in the measured intensities was estimated by applying Cruickshank's formula (Cruickshank, 1949). The r.m.s. average of the estimated standard deviations for the coordinates was approximately the same for all directions and equal to approximately $0.014 \AA$.

Attempts were also made to estimate the errors due
Table 3. $\mathrm{P}_{4} \mathrm{~S}_{7}$ : final atomic coordinates

|  | $x$ | $y$ | $z$ |
| :--- | ---: | :---: | :---: |
| $\mathbf{P}_{1}$ | 0.281 | 0.379 | 0.709 |
| $\mathbf{P}_{2}$ | 0.353 | 0.293 | 0.463 |
| $\mathbf{P}_{3}$ | 0.271 | 0.461 | 0.274 |
| $\mathbf{P}_{4}$ | 0.003 | 0.335 | 0.409 |
| $\mathrm{~S}_{1}$ | 0.043 | 0.425 | 0.220 |
| $\mathbf{S}_{2}$ | 0.307 | 0.544 | 0.094 |
| $\mathbf{S}_{3}$ | -0.212 | 0.308 | 0.363 |
| $\mathbf{S}_{4}$ | 0.305 | 0.487 | 0.569 |
| $\mathbf{S}_{5}$ | 0.413 | 0.367 | 0.239 |
| $\mathbf{S}_{6}$ | 0.150 | 0.245 | 0.366 |
| $\mathbf{S}_{7}$ | 0.052 | 0.366 | 0.697 |

to an imperfect correction for series termination by comparing the backshifts obtained from $F_{c}$ syntheses of the [001] projection, based on slightly different

[^2]
(a)

(b)

Fig. 6. $\mathrm{P}_{4} \mathrm{~S}_{7}$ : projection (a) along [100] on the (100) plane, (b) along [001] on the (001) plane.
coordinates. These coordinates were all close to the final ones. Two of such comparisons were made, the r.m.s. average $\left\{\overline{(\Delta q)^{2}}\right\}^{\frac{1}{2}}$ of the differences of the coordinates being $0.02_{9}$ and $0.01_{4} \AA$ respectively. In both cases the mean backshifts obtained, $\left(\overline{b^{2}}\right)^{\frac{1}{2}}$, are approximately the same and equal to $0 \cdot 02_{5} \AA$ for the two $\boldsymbol{F}_{c}$ syntheses involved; the backshift for each individual coordinate, however, appeared in general to be different in both $F_{c}$ syntheses. The r.m.s. average $\left\{\overline{(\Delta b)^{2}}\right\}^{\frac{1}{2}}$ of these backshift differences are $0.01_{1}$ and $0.007 \AA$ for the two sets of $F_{c}$ syntheses respectively, being almost one-half of the mean differences, $\left\{\overline{(\Delta q)^{2}}\right\}^{\frac{1}{2}}$, of the coordinates on which the corresponding set of $\boldsymbol{F}_{c}$ syntheses were based. This means that the backshift corrections involve an inaccuracy, even if these corrections are taken from a $F_{c}$ synthesis based on the 'final' coordinates. Denoting the standard deviation of the coordinates due to inaccuracies in the intensities by $\sigma_{i}$ and that due to the imperfect backshift correction by $\sigma_{b}$, the combined standard deviation $\sigma_{i b}$ is given by $\sigma_{i b}=\left(\sigma_{i}^{2}+\sigma_{b}^{2}\right)^{\frac{1}{2}}$ if it is assumed that no direct correlation exists between the two sources of error. When $\sigma_{i}$ is known, $\sigma_{i b}$ may be calculated from

$$
\sigma_{b}=f \sigma_{i b}=f\left(\sigma_{i}^{2}+\sigma_{b}^{2}\right)^{\frac{1}{2}} \text { or } \sigma_{i b}=\left(1-f^{2}\right)^{-\frac{1}{2}} \sigma_{i}
$$

in which $f \equiv\left\{\overline{(\Delta b)^{2}}\right\}^{\frac{1}{2}} /\left\{\overline{(\Delta q)^{2}}\right\}^{\frac{1}{2}}$.
With $\sigma_{i}=0.014 \AA$ and $f=0.5$ a value of $0.016 \AA$ for $\sigma_{i b}$ is calculated. With this standard deviation in the coordinates, a standard deviation of $\left\{(0.016 / 2)^{2}+\right.$ $\left.0.01^{2}\right\}^{\frac{1}{2}}=0.025 \AA$ in the bond lengths is obtained when the error due to inaccuracies in the cell dimensions is represented by a standard deviation of $0.01 \AA$ for bond lengths of approximately $2 \AA$. The corresponding standard deviation in the bond angles is $1.0^{\circ}$ (Cruickshank \& Robertson, 1953).

## Discussion

The structure consists of molecules of $\mathrm{P}_{4} \mathrm{~S}_{7}$ separated by shortest intermolecular distances of approximately $3 \cdot 4 \AA$. The [100] and [001] projections are shown in Fig. 6. The general shape of the molecule (Fig. 7) does not correspond to any of the structures suggested for $\mathbf{P}_{4} \mathrm{~S}_{7}$ previously; from the valencies of phosphorus and sulfur, it was concluded that apart from two phos-


Fig. 7. Molecule of $P_{\mathbf{4}} \mathrm{S}_{\mathbf{7}}$.
Table 4. $\mathrm{P}_{4} \mathrm{~S}_{7}$ : intramolecular bond lengths and bond angles

| $\mathrm{P}_{1}-\mathrm{P}_{2}$ | $2.37 \AA$ | $\mathrm{~S}_{2}-\mathrm{P}_{3}-\mathrm{S}_{1}$ | $107^{\circ}$ | $\mathrm{P}_{3}-\mathrm{S}_{5}-\mathrm{P}_{2}$ | $103^{\circ}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{P}_{1}-\mathrm{S}_{4}$ | 2.12 | $\mathrm{~S}_{3}-\mathrm{P}_{4}-\mathrm{S}_{1}$ | 106 | $\mathrm{P}_{4}-\mathrm{S}_{7}-\mathrm{P}_{1}$ | 103 |
| $\mathrm{P}_{1}-\mathrm{S}_{7}$ | 2.04 | $\mathrm{~S}_{2}-\mathrm{P}_{3}-\mathrm{S}_{4}$ | 116 | $\mathrm{P}_{4}-\mathrm{S}_{6}-\mathrm{P}_{2}$ | 101 |
| $\mathrm{P}_{2}-\mathrm{S}_{5}$ | 2.08 | $\mathrm{~S}_{2}-\mathrm{P}_{3}-\mathrm{S}_{5}$ | 113 | $\mathrm{P}_{3}-\mathrm{S}_{1}-\mathrm{P}_{4}$ | 108 |
| $\mathrm{P}_{2}-\mathrm{S}_{6}$ | 2.06 | $\mathrm{~S}_{3}-\mathrm{P}_{4}-\mathrm{S}_{7}$ | 112 | $\mathrm{~S}_{4}-\mathrm{P}_{1}-\mathrm{P}_{2}$ | 102 |
| $\mathrm{P}_{3}-\mathrm{S}_{1}$ | 2.13 | $\mathrm{~S}_{3}-\mathrm{P}_{4}-\mathrm{S}_{6}$ | 114 | $\mathrm{~S}_{5}-\mathrm{P}_{2}-\mathrm{P}_{1}$ | 103 |
| $\mathrm{P}_{3}-\mathrm{S}_{2}$ | 1.93 | $\mathrm{~S}_{4}-\mathrm{P}_{3}-\mathrm{S}_{1}$ | 110 | $\mathrm{~S}_{7}-\mathrm{P}_{1}-\mathrm{P}_{2}$ | 101 |
| $\mathrm{P}_{3}-\mathrm{S}_{4}$ | 2.08 | $\mathrm{~S}_{5}-\mathrm{P}_{3}-\mathrm{S}_{1}$ | 109 | $\mathrm{~S}_{6}-\mathrm{P}_{2}-\mathrm{P}_{1}$ | 103 |
| $\mathrm{P}_{3}-\mathrm{S}_{5}$ | 2.09 | $\mathrm{~S}_{7}-\mathrm{P}_{4}-\mathrm{S}_{1}$ | 111 | $\mathrm{~S}_{4}-\mathrm{P}_{1}-\mathrm{S}_{7}$ | 102 |
| $\mathrm{P}_{4}-\mathrm{S}_{1}$ | 2.07 | $\mathrm{~S}_{6}-\mathrm{P}_{4}-\mathrm{S}_{1}$ | 111 | $\mathrm{~S}_{5}-\mathrm{P}_{2}-\mathrm{S}_{6}$ | 105 |
| $\mathrm{P}_{4}-\mathrm{S}_{3}$ | 1.97 | $\mathrm{~S}_{4}-\mathrm{P}_{3}-\mathrm{S}_{5}$ | 103 |  |  |

phorus atoms, each surrounded by four sulfur atoms, the molecule contains two directly linked phosphorus atoms at a distance of $2 \cdot 37 \AA$. The distances $P_{3}-S_{2}$ and $\mathrm{P}_{4}-\mathrm{S}_{3}$ are 1.93 and $1.97 \AA$ respectively. The other P-S distances range from 2.04 to $2.13 \AA$ (Table 4). A many-parameter significance test (Cruickshank \& Robertson, 1953) showed these observations to be in agreement with the assumption of only two different types of bonds in the molecule: 'double' bonds, between $P_{3}-S_{2}$ and $P_{4}-S_{3}$, and 'single' bonds, the lengths of these bonds being equal to the average values of $1 \cdot 95_{0}$ and $2 \cdot 08_{0} \AA$ respectively. In the significance test a value of $T_{o}^{2}=12 \cdot 1$, corresponding to $P=0.2$ for ten degrees of freedom, was obtained.

The following types of bond angles may be distinguished in the molecule: $\mathrm{S}_{2} \mathrm{P}_{3} \mathrm{~S}_{1}, \mathrm{~S}_{2} \mathrm{P}_{3} \mathrm{~S}_{4}, \mathrm{~S}_{4} \mathrm{P}_{3} \mathrm{~S}_{1}$, $\mathrm{S}_{4} \mathrm{P}_{3} \mathrm{~S}_{5}, \mathrm{P}_{3} \mathrm{~S}_{4} \mathrm{P}_{1}, \mathrm{P}_{3} \mathrm{~S}_{1} \mathrm{P}_{4}, \mathrm{~S}_{4} \mathrm{P}_{1} \mathrm{P}_{2}$ and $\mathrm{S}_{4} \mathrm{P}_{1} \mathrm{~S}_{7}$. The average value for these angles is $106 \cdot 6,113 \cdot 7,110 \cdot 1,103 \cdot 0$, $102 \cdot 8,107 \cdot 8,102 \cdot 2$ and $103 \cdot 6^{\circ}$ respectively, the largest deviations from the mean value being twice the standard deviation. Consequently the mutual differences between the individual bond lengths and angles of the same type do not indicate a significant deviation from a symmetry $m m 2$ of the molecule. In this molecule the $\mathrm{PS}_{4}$ tetrahedra are distorted since the average bond angles of the types $\mathrm{S}_{2} \mathrm{P}_{3} \mathrm{~S}_{1}, \mathrm{~S}_{2} \mathrm{P}_{3} \mathrm{~S}_{4}$ and $\mathrm{S}_{4} \mathrm{P}_{3} \mathrm{~S}_{5}$ show a significant deviation from $109.5^{\circ}$.

The average values for the two P-S bond types, namely $2 \cdot 08_{0}$ and $1 \cdot 95_{0} \AA$ with an estimated standard deviation of 0.01 and $0.02 \AA$ respectively, agree very well with the corresponding values $\left(2.08_{5}\right.$ and $1.95_{5} \AA$ respectively) in the $\mathrm{P}_{4} \mathrm{~S}_{10}$ molecule. The standard deviation of the atomic distance $\mathrm{P}_{1}-\mathrm{P}_{2}$ is somewhat larger than $0.02_{5} \AA$, as these atoms are not very well resolved in the projections, and was estimated to be $0.04 \AA$. The P-P distance, $2 \cdot 37 \AA$, is remarkably large in comparison with P-P distances known from other structures. In black phosphorus the shortest P-P distances are $2 \cdot 18 \AA$ (Hultgren, Gingrich \& Warren, 1935) and $P_{4}$ tetrahedra in the vapour phase show a P-P distance of $2 \cdot 21 \AA$ (Maxwell, Hendricks \& Mosley, 1935).

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[^0]:    * A table of observed and calculated structure factors has been deposited as Document No. 4489 with the ADI Auxiliary, Publications Project, Photoduplication Service, Library of Congress, Washington 25, D.C., U.S.A. A copy may be secured by citing the Document number and by remitting $\$ 1.25$ for photoprints, or $\$ 1.25$ for 35 mm . microfilm. Advance payment is required. Make cheques or money orders payable to: Chief, Photoduplication Service, Library of Congress.

[^1]:    * See footnote p. 218.

[^2]:    * See footnote on p. 218.

